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THE STATIC THEORY OF TRANSFER PRICING

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ABSTRACT

An analysis of the literature of transfer pricing is presented. It is shown that, under assumptions that the firm and its divisions have full deterministic knowledge of their costs and demands, some form of average cost is the appropriate transfer price. What happens when a firm adopts an objective other than profit maximization is further examined.

1. INTRODUCTION

It has been claimed that four-fifths of all major U.S. corporations are largely or wholly decentralized [2]. If true this implies that most top managers must believe that the advantages of decentralization exceed the advantages of centralization in large corporations. There is little doubt that when decentralization is established with care, commitment, and knowledge, it is easier to evaluate the relative performance of a division and its manager in terms of economic criteria than under centralization. One of the ingredients of a successfully decentralized firm is a system of transfer prices at which products are transferred between the divisions of the firm that guides the divisions to accomplish the objectives of the firm through suboptimization. Since we know that suboptimizations by individual divisions of a firm do not necessarily result in an optimum for the firm as a whole, the transfer pricing problem consists in fixing these prices in such a way that divisional suboptimizations imply an overall optimum. Several authors [3, 9] have stated that this problem is inherently insoluble because firms have multiplicities of objectives and it is not possible to design a system of transfer prices which attains *all* objectives of a firm optimally. This may very well be the case. At any rate, although transfer prices are only one ingredient among many in decentralization, Adam Smith's invisible hand will not guide the parts toward the optimum of the whole unless considerable care is exercised in setting prices.

A variety of pricing policies are followed in practice. Products are transferred at average cost, average variable cost, variable cost plus a margin, marginal cost, market price, market price minus a margin, "long-run value," market-based negotiated price, and some intermediate forms adapted to particular situations. The theoretical basis of transfer pricing was provided by Hirshleifer [12, 13] who showed in two articles that under static conditions and under certainty the generally correct transfer price for a product is the marginal cost of the producing division. Hirshleifer made the usual simplifying assumptions, one of them being that the firm pursues the single objective of profit maximization. His solution has been accepted as the theoretically correct solution by apparently all authors on transfer pricing, and has become the basis of further extensions and elaborations. It is the purpose of this article to show that Hirshleifer's solution is incorrect. The correct generally valid transfer price under the same conditions as assumed by Hirshleifer is not marginal cost, but a form of average cost. Although transfers at marginal cost if possible will bring about correspondence between the suboptima

of the divisons and the optimum of the firm, they are neither theoretically correct nor very desirable as pricing rules. Aside from the usual problem of determining marginal costs in practice, when the optimum point lies on the increasing branch of a marginal cost curve the buying division will consider the transfer of the product at marginal cost as inequitable. And when the point lies on the decreasing branch, the selling division will be unhappy. Moreover, as we shall see, marginal cost pricing has to be accompanied by a stricture, already mentioned by Hirshleifer, which practically means that the buying division is told what quantity it should buy from the selling division. But this largely defeats the purpose of decentralization.

We will show in the following sections that for the typical cases discussed by Hirshleifer and others, some form of average cost is the appropriate transfer price. When constraints are included, the average cost has to be adjusted in terms of opportunity profits. We will further examine what happens when a firm adopts an objective other than profit maximization. It is assumed that the firm and its divisions have full deterministic knowledge of their costs and demands.

2. THE HIRSHLEIFER ANALYSIS

The following assumptions are made. A firm establishes two profit centers; i.e., the objective of each center is to maximize its respective profit. The objective of the firm as a whole is also profit maximization. The first division, to be called Division 1, is a manufacturing division whose output is transferred to Division 2, a distribution division, which finishes the product and sells it in the market. We shall confine ourselves in this section to the case when the output of Div. 1 does not have a market price. In other words, Div. 1 produces an intermediate product which is not traded in the market. Division 2 finishes the intermediate product and sells the final product at the prevailing market price, which we suppose is constant and given to the firm. Since each division operates as a profit center, the output of Div. 1 will be charged to Div. 2 and the problem of the firm is to fix a transfer price so that the independent actions of the two divisions will maximize the overall profit of the firm.

To simplify the analysis, we assume that there exists no technological dependence between the two divisions in the sense that the costs of one division are independent of the activity levels of the other divsion except for the costs of the intermediate product. This means that there is no joint-product relationship between the two divisions in the technical sense. Since our approach is static, inventory accumulation, depreciation of equipment, and other dynamic aspects can be ignored. To determine the output level of Div. 2 which maximizes the profit of the firm (remember, only Div. 2 operates in the market), we perform the usual operation of finding that output at which the marginal cost of the firm, MC, equals the market price, p,

$$MC=p.$$

The marginal cost of the firm is the sum of the marginal costs of the divisions,

$$MC=MC_1+nMC_2,$$

where nMC_2 (net marginal cost) excludes the transfer price of the intermediate product. If we assume that the units of the intermediate and final products are commensurate in the sense that both have a specific quantitative relationship to each other, the equation

$$p = MC_1 + nMC_2$$

can then be solved for the optimum output levels.

Hirshleifer solves the problem of optimum outputs and the appropriate transfer price with the help of a diagram, Figure 1. Both the output of Div. 1, designated by u, and the output of Div. 2, designated by q, are measured along the same axis since commensurability between the two outputs is assumed. The optimum output for Div. 2, denoted by q^0, occurs where the rising marginal cost curve of the firm intersects the price line p. This output level can be reached through the suboptimizations of the divisions as follows. Div. 2 obtains from the manufacturing division a schedule showing what quantity of the intermediate product the manufacturing division would produce—and sell—at any transfer price r_u. This schedule would be the same as the MC_1 curve if the manufacturing division rationally determines its output and sets $MC_1 = r_u$. With this information the distribution division can derive a curve, labelled $p - r_u$ in the diagram, which represents the difference between market price and transfer price for any level of output. The distribution division then finds its output level by operating at the point where the "net marginal revenue" curve $p - r_u$ intersects nMC_2. The manufacturing division also produces at the same level because $MC_1 = r_u$ there. Hirshleifer then concludes that the transfer price should equal the marginal cost of the manufacturing division for the level q^0.

This solution causes several problems. First, for the case of Fig. 1, Div. 1 makes a "profit" represented by the area between the curve MC_1 and the line $r_u^0 r_u^0$. "Profit" is made whenever the solution lies on the rising branch of the marginal cost curve since the average cost curve is below the marginal curve. What happens, however, when the solution occurs on the downward sloping branch of the marginal cost curve as is frequently the case? The average cost curve then lies above the marginal curve and by setting the transfer price equal to marginal cost, Div. 1 would record a "loss." A transfer pricing system that is not equitable in the eyes of both divisions would have little chance accomplishing its goals. Second, the marginal cost pricing rule must be accompanied by a stipulation.(1) Division 2 must not be allowed to increase its profit by operating on a curve marginal to the curve $p - r_u$ (usually called a quasi-marginal curve in economics), the curve labelled mr in Fig. 1. If Div. 2 were permitted to move to the point where $mr = nMC_2$, it would sell the quantity q^* instead of the optimal q^0, and thereby increase the profit of Div. 2, but decrease the overall profit of the firm.(2) Division 2's profit for the output q^0 is represented by the area between the line segment ss and the nMC_2 curve. For the output level q^* the profit becomes the area between the line segment tt and the corresponding segment of nMC_2.

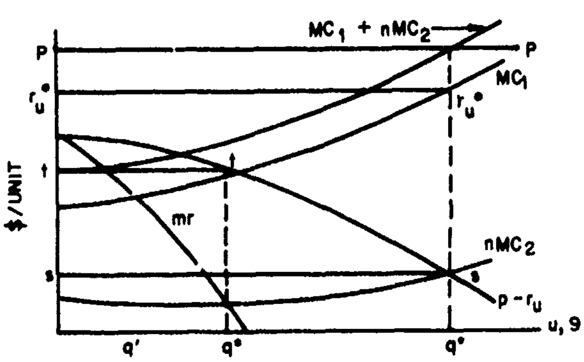

FIGURE 1

Since Div. 2's profit is larger at level q^* than at q^0, and since the curves are all continuous, Div. 2 can increase its profit by choosing any output level in the interval q^*q^0. The marginal cost rule must therefore be accompanied by the requirement that Div. 2 cannot operate in the interval q^*q^0, because any move away from q^0 toward q^* will reduce the profit of the firm. But this is not enough. A certain output interval to the left of q^*, say $q'q^*$, must also be declared off limits to Div. 2 since there exists a neighborhood to the left of q^* in which profits to Div. 2 are greater than at q^0. (Again, with continuous curves profit rises and declines gradually.) It follows that after we eliminate the interval $q'q^*$, Div. 2 as a profit maximizer is indifferent between outputs q' and q^0, and to make sure that the global optimum level q^0 is selected, we also stipulate that Div. 2 may not operate in the interval $0q'$. But this in effect means that we are telling Div. 2 to sell the quantity q^0 and the purpose of decentralized decision making has been largely defeated.

Third, for such decisions as whether or not to shut down a division, marginal rules which by definition represent values of incremental or decremental changes are of little use and one has to go back to total magnitudes.

Although practice has shown that there does not seem to exist a transfer price which cannot be faulted in some way, the relatively serious first and second drawbacks mentioned should have suggested that there may be something wrong with the marginal cost pricing rule.[3] We will now work out the correct transfer price under the same assumptions as for the Hirshleifer analysis.

3. NO MARKET FOR THE INTERMEDIATE PRODUCT

We shall employ a few relatively simple mathematical relations and manipulations. Assume that Div. 1 uses two inputs, x_1 and x_2, to produce the intermediate output u, and that inputs and outputs are related by an economic production function,

$$u = f(x_1, x_2).$$

This function summarizes (and suppresses many details important in real production) the generally rather complex production relationships in terms of the three stated variables. We shall suppose that this function and all other mathematical terms to be introduced are continuous and can be differentiated. Let us further designate the unit cost of input i by r_i $(i = 1, 2)$ and treat it as a constant. The as yet unknown transfer price of the intermediate product will be denoted by r_u. Then if Div. 1 is a profit center, its problem is:

$$\text{maximize } \pi_1 = r_u u - r_1 x_1 - r_2 x_2$$

$$\text{subject to the production function } u = f(x_1, x_2),$$

where $r_u u$ represents the receipts of the division, $r_1 x_1 + r_2 x_2 = C_1$ its costs, and the symbol π profits.

(1) Hirshleifer (1956, p. 175)

(2) The reader who wishes to see a numerical demonstration of the increase in profit, may consult Naert and Janssen who have worked out a simple example.

(3) The disadvantages of the marginal cost pricing rule have been recognized by many authors, Hirshleifer himself being one of the first. Particularly, Ronen and McKinney, Shillinglaw, and Solomons have discussed marginal cost pricing in detail and shown its defects.

We may adopt similar relations for Division 2. Suppose that Division 2 also uses two inputs, x_3 and u, to produce the final product q. The production function is

$$q=g(x_3, u),$$

showing that the output of Division 1 becomes an input to Division 2. We denote the unit price of input x_3 by r_3 and the market price of the final product by p, and suppose that both are constant. Since Division 2 also operates as a profit center, its problem is:

$$\text{maximize } \pi_2=pq-r_3x_3-r_uu$$

$$\text{subject to the production function } q=g(x_3, u).$$

Finally we need to formulate the problem of the firm. Since Division 2 only sells in the market, the receipts of the firm are identical with the receipts of Division 2. To determine profit, we subtract total costs $C=r_1x_1+r_2x_2+r_3x_3$ from the receipts. The problem of the firm is therefore:

$$\text{maximize } \pi=pq-r_1x_1-r_2x_2-r_3x_3$$

$$\text{subject to the production functions } u=f(x_1, x_2) \text{ and } q=g(x_3, u).$$

Notice that the transfer price r_u does not appear in the problem of the firm because the cost of u to the firm is already taken care of by the term $r_1x_1+r_2x_2$.

Let us formulate the corresponding Lagrangian functions for the three problems:

$$L_1=r_uu-r_1x_1-r_2x_2-\lambda_1[f(x_1, x_2)-u],$$

$$L_2=pg(x_3, u)-r_3x_3-r_uu, \text{ and}$$

$$L=pg(x_3, u)-r_1x_1-r_2x_2-r_3x_3-\lambda[f(x_1, x_2)-u],$$

where λ_1 and λ represent Lagrangian multipliers. In order to find the optimum solutions to the three problems, we need to differentiate each Lagrangian function in terms of its variables and set each of the partial derivatives equal to zero:

Div. 1

$$\begin{aligned}
\partial L_1/\partial u &= \partial(r_uu)/\partial u+\lambda_1 = 0\\
\partial L_1/\partial x_1 &= -r_1-\lambda_1 f_1 = 0\\
\partial L_1/\partial x_2 &= -r_2-\lambda_1 f_2 = 0\\
\partial L_1/\partial \lambda_1 &= f-u = 0
\end{aligned}$$

Div. 2

$$\begin{aligned}
\partial L_2/\partial x_3 &= pg_3-r_3 = 0\\
\partial L_2/\partial u &= pg_u-\partial(r_uu)/\partial u = 0
\end{aligned}$$

Firm

$$\begin{aligned}
\partial L/\partial x_3 &= pg_3-r_3 = 0\\
\partial L/\partial u &= pg_u+\lambda = 0\\
\partial L/\partial x_1 &= -r_1-\lambda f_1 = 0\\
\partial L/\partial x_2 &= -r_2-\lambda f_2 = 0\\
\partial L/\partial \lambda &= f-u = 0
\end{aligned}$$

where $f_i = \partial f/\partial x_i$, $g_u = \partial g/\partial u$, etc. As is well known, these conditions are only necessary for a maximum, but we shall not be concerned with second-order conditions here.

Let us see what needs to be done now. The firm's conditions consist of five equations and five variables (x_1, x_2, x_3, u, λ) and therefore, in principle, can be solved for the optimum values which will maximize the profit of the firm. We want to establish a correspondence between the equations of the firm and the equations of the divisions so that the optimum solutions to the divisional problems also solve the problem of the firm.

Before we do this we need to know what the Lagrangian multipliers stand for. If $\partial L_1/\partial x_1$ is multiplied by dx_1, $\partial L_1/\partial x_2$ by dx_2, and both are added, we get

$$dC_1 = r_1 dx_1 + r_2 dx_2 = -\lambda_1 (f_1 dx_1 + f_2 dx_2)$$
$$= -\lambda_1 du$$

and $-\lambda_1 = dC_1/du$. In words: The Lagrangian multiplier times (-1) represents the marginal cost of Div. 1. By performing the same operations with $\partial L/\partial x_1$ and $\partial L/\partial x_2$, we also get $-\lambda = dC_1/du$.

Now observe that the last three equations of the firm's first-order conditions agree with the last three equations of Div. 1. Moreover, $\partial L/\partial x_3$ is the same as $\partial L_2/\partial x_3$, and all we have to do is to establish a correspondence between $\partial L_1/\partial u$, $\partial L_2/\partial u$, and $\partial L/\partial u$. But this can easily be done by setting

$$\partial (r_u u)/\partial u = -\lambda_1 = -\lambda = MC_1 = dC_1/du$$

for then $\partial L_1/\partial u$ is identically satisfied, and Div. 2 will demand input u until the marginal revenue product pg_u equals the marginal cost of producing u, the same requirement imposed by the condition $\partial L/\partial u = 0$. In other words, with this choice the firm's optimal values x_1^0, x_2^0, x_3^0, u^0, λ^0 also satisfy the six divisional equations. Or, remembering the purpose of decentralization, when $\partial (r_u u)/\partial u = MC_1$, each division acting independently will operate at a level which maximizes the overall profit of the firm. For a specified value of u, Div. 1 determines the optimal values[4] of x_1, x_2, λ_1, while Div. 2 establishes the optimal x_3 and u. Thus, for given p and r_3, Div. 2 demands from Div. 1 the amount u^0 of the intermediate product, and Div. 1 in turn produces u^0 with the optimal input mix x_1^0, x_2^0.

How about the transfer price? If we integrate the last equation, we get

$$r_u u = \int \frac{dC_1}{du}\, du + k$$

$$= C_1(u) + k$$

and the transfer price becomes

$$r_u = [C_1(u) + k]/u,$$

where k is an integration constant. Since $C_1(u)$ represents the total variable cost of Div. 1, we conclude that the transfer price equals the average variable cost of Div. 1, $C_1(u)/u$, plus (or minus) the term k/u. If k is set equal to total fixed cost, the transfer price will be the average cost. For most applications

[4] Since $\partial L_1/\partial u$ is an identity, Div. 1 has only three independent conditions.

average cost would seem to be the most appropriate pricing rule.[5] But since k is arbitrary, it can assume other values than total fixed cost if the situation requires it and still lead the divisions to the correct theoretical solution.[6] Thus, we could assign k a value of zero and transfer at average variable cost.[7] Because the constant k is arbitrary, we cannot speak of *the* transfer price, but merely of *a* transfer price. But in any event, the transfer price is some sort of an average. Why then did Hirshleifer determine MC_1 as the transfer price? Because he implicitly assumed that r_u does not depend on the output level of the intermediate product but is a constant. Note that when r_u is taken as a constant, we get $\partial(r_u u)/\partial u = r_u = MC_1$, the Hirshleifer solution. Can r_u be a constant? Yes, when, e.g., the production function $f(x_1, x_2)$ has the property of being homogeneous of degree 1. In this case the marginal cost of Div. 1 is a horizontal line and coincides with average variable cost. But this is only a special situation and the solution is not valid generally.[8] In particular, it is not valid for the Hirshleifer analysis because, when we examine his diagram once more, we see that he has drawn MC_1 as an increasing curve implying that marginal cost, and therefore r_u, depends on the output level.

Notice that when, e.g., r_u = average cost, we have no problems in the case of a declining marginal cost curve since average cost lies above marginal cost. Nor is there any necessity to restrict the output of a division as with the Hirshleifer solution. Division 2, being responsible for the total "profit" (except, perhaps, for a constant deduction k), has no incentive to search for a quasi-marginal revenue curve. The optimum output level for the firm is also the optimum for Division 2.

The result that the transfer price equals the average variable cost plus or minus k/u, is the general theoretical solution of the transfer pricing problem. It applies when there are more than two inputs and more than two divisions and, as we shall see in the next section, when there is a market for the intermediate product.

4. MARKET FOR THE INTERMEDIATE PRODUCT

We will assume that demand between the intermediate and the final product is independent, i.e., the volume of sales of one product does not affect the price of the other product. Let us first consider the case when the market prices of both products are constant. We designate the market price of the intermediate product by m, and use the equation $u = u_1 + u_2$ to indicate that of the total output u of the manufacturing division, u_1 is sold in the market and u_2 transferred to Division 2. The transfer price

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(5) Ronen and McKinney also have suggested that average cost should be the appropriate transfer price. They adopt it to overcome the deficiencies of marginal cost pricing although they still hold that Hirshleifer's solution is the correct solution.

(6) One reader of this paper has suggested that the solution $r_u = [C_1(u) + k]/u$ is trivial since it is not *the* solution but allows many solutions by the arbitrariness of the constant k. That is a misunderstanding. What is relatively trivial is the mathematics used to derive the stated result. There is nothing very profound about an integration constant. But the fact that one appears does not render the result trivial. The constant simply tells us that we can adjust the transfer price to fit different circumstances and still have a correct solution.

(7) The Department of the Air Force several years ago decided to experiment with transfer pricing by requiring an Air Force test center to operate like an independent firm to the extent that this is possible in such an environment. The adjustment was a painful process as one would expect when an organization that had always been institutionally funded all of a sudden is ordered to act like a competitive firm. For a variety of reasons the experiment was not successful. One reason was that the facility was required to transfer close to average cost. Since a test facility has high fixed costs, it probably would have been more reasonable to initially require that transfers be priced at average variable cost until the managers and employees had become used to a radically different way of doing business.

(8) By casting the problem into a linear programming form, several authors have shown that marginal cost is the appropriate transfer price. But, of course, a linear programming formulation implies a linear homogenous production function.

is now designated by r_{u_2}. If otherwise the same relations and symbols are retained as before, we have the Lagrangian functions

$$L_1 = mu_1 + r_{u_2}u_2 - r_1x_1 - r_2x_2 - \lambda_1[f(x_1, x_2) - u] - \theta_1[u_1 + u_2 - u]$$

$$L_2 = pg(x_3, u_2) - r_3x_3 - r_{u_2}u_2$$

$$L = pg(x_3, u_2) + mu_1 - r_1x_1 - r_2x_2 - r_3x_3 - \lambda[f(x_1, x_2) - u] - \theta[u_1 + u_2 - u]$$

from which three sets of first-order conditions can be derived. Proceeding as in the last section, we obtain

$$r_{u_2} = m + \frac{k}{u_2} = \frac{[C_1(u_1 + u_2) + \phi(u_1)]}{u_2},$$

as the transfer price, where k and $\phi(u_1)$ are integration constants. The transfer price equals the market price plus or minus a term k/u_2. No purpose seems to be served in letting k be positive or negative. It would only introduce frictions between the divisions. The solution is therefore the same as Hirshleifer's: the transfer price is set equal to the market price of the intermediate product.

This solution does not change when the market price of the final product is no longer a constant. But when the price of the intermediate product also becomes a variable, the transfer pricing problem is more complicated. The transfer price is still the same but a division can no longer determine its optimum output without knowing the optimum output of the other division. This is obviously a problem of mutual dependence and the firm may prefer to have both divisions under single management.

5. TRANSFER PRICES IN THE PRESENCE OF CONSTRAINTS

Let us first consider a case where the head office for whatever reason wants the output of the manufacturing division not to be less than a certain quantity a^*, i.e., $u \geq a^*$. If we treat Div. 1 as a cost center, suppose that the intermediate product is not traded in the market, and take the final product price as a constant, we have the following objectives:

Div. 1 . . . minimize $C_1 = r_1x_1 + r_2x_2$
subject to $u = f(x_1, x_2)$

Div. 2 . . . maximize $\pi_2 = pq - r_3x_3 - r_uu$
subject to $q = g(x_3, u)$

Firm . . . maximize $\pi = pq - r_1x_1 - r_2x_2 - r_3x_3$
subject to $u = f(x_1, x_2)$, $q = g(x_3, u)$, and $u \geq a^*$.

We could, of course, impose the constraint $u \geq a^*$ directly on Div. 1, but unless we force Div. 2 in a similar way, 2 may buy a smaller quantity than a^* from 1. And the purpose of decentralization is to attain by the appropriate choice of transfer prices the objectives of the firm through suboptimization.

The corresponding Lagrangian functions for the above objectives are:

$$L_1 = r_1x_1 + r_2x_2 - \lambda_1[f(x_1, x_2) - u]$$

$$L_2 = pg(x_3, u) - r_3x_3 - r_uu$$

$$L = pg(x_3, u) - r_1x_1 - r_2x_2 - r_3x_3 - \lambda[f(x_1, x_2) - u] - \theta[u - a^* - s^2].$$

The constraint $u \geqslant a^*$ has been put in the form $u = a^* + s^2$ and incorporated into the function of the firm. The slack variable s^2 ensures that u cannot be less than a^*. As before, we derive the first-order conditions which, upon comparison, show that the transfer price is

$$r_u = \left[C_1(u) + \int \frac{d\pi}{da}\, du + k \right] / u.$$

To get a better understanding of the additional term $\int \frac{d\pi}{da}\, du$, consider Fig. 2, which shows the firm's profit as a function of the parameter a. It is assumed that the optimal u would be less than a^* if the constraint $u \geqslant a^*$ were eliminated. The level a^0 is obtained by setting $a^0 = u^0$, the optimal output without the constraint. When a^* does not exceed a^0, u would be set equal to the optimal a^0 and $\theta = d\pi/da = 0$ — the profit curve has zero slope over the interval $(0, a^0)$. When $a^* > a^0$, $\theta = d\pi/da < 0$ and $\int_{a^0}^{a^*} (d\pi/da)\, du$ represents the opportunity profit the firm gives up by requiring the manufacturing division to produce, and the distribution division to use, the amount $u = a^*$ instead of the optimal $u = a^0$. The integral is therefore negative and subtracted from the costs of Div. 1,

$$\int_{a^0}^{a^*} \frac{d\pi}{da}\, du = \pi(a^*) - \pi(a^0) < 0.$$

The opportunity profit function $\pi(a)$ can be easily computed from the first-order conditions of the firm. We conclude that the transfer price equals average cost less average opportunity profit,

$$r_u = \frac{C_1(u) + k}{u} - \frac{\pi(a^0) - \pi(a^*)}{u}.$$

Because of the lower transfer price to Div. 2 (lower relative to charging average cost), Div. 2 will demand a larger quantity of u (relative to a^0). More precisely, with this transfer price the amount a^*

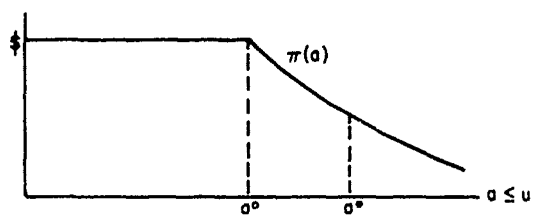

FIGURE 2

will be demanded by Div. 2 and without any direct constraints having been imposed on the divisions. Since Div. 1 is treated as a cost center, the head office may credit it with the opportunity profit $\pi(a^\circ)-\pi(a^*)$.

It follows that for the type of constraint we have been discussing, the transfer price is obtained by subtracting (or adding, as the case may be) the average opportunity profit from average cost.

Average cost can be adjusted in different ways, though, as we see by examining a case when the constraint is a budget constraint of the form $C=r_1x_1+r_2x_2+r_3x_3 \leq \bar{C}$, a constant. Let us suppose once again that the intermediate product is traded in the market. We shall assume that the firm's total costs would exceed the amount $\bar{C}$ for an optimal solution if there were no budget constraint. This eliminates the necessity of adding a slack variable to the inequality $C \leq \bar{C}$ as cost then equals the bound $\bar{C}$. From the appropriate Lagrangian functions, we get the following first-order conditions:

Div. 1	*Div. 2*	*Firm*
$\frac{\partial(mu_1)}{\partial u_1}-\theta_1=0$	$pg_3-r_3=0$	$pg_3-r_3(1+\eta)=0$
$\frac{\partial(r_{u_2}u_2)}{\partial u_2}-\theta_1=0$	$pg_{u_2}-\frac{\partial(r_{u_2}u_2)}{\partial u_2}=0$	$pg_{u_2}-\theta=0$
		$\frac{\partial(mu_1)}{\partial u_1}-\theta=0$
$-r_1-\lambda_1f_1=0$		$-r_1(1+\eta)-\lambda f_1=0$
$-r_2-\lambda_1f_2=0$		$-r_2(1+\eta)-\lambda f_2=0$
$f-u=0$		$f-u=0$
$u_1+u_2-u=0$		$u_1+u_2-u=0$
$\lambda_1+\theta_1=0$		$\lambda+\theta=0$
		$r_1x_1+r_2x_2+r_3x_3-\bar{C}=0$

Now $\eta=d\pi/d\bar{C}$, i.e., the Lagrangian multiplier η equals the change of profit per unit change in the budget. If the optimal C is larger than C without the constraint, η is positive: profit increases with an increasing budget. The magnitude of η can be determined from the first-order conditions of the firm.

Comparing the firm's conditions with the divisional conditions, we observe that if the divisions are required to use unit *standard* input costs of $(1+\eta)r_1$, $(1+\eta)r_2$, and $(1+\eta)r_3$ instead of r_1, r_2, and r_3, there is agreement between the global solution and the divisional solutions. Thus,

$$-\lambda_1=\theta_1=-\lambda=\theta=(1+\eta)MC_1,$$

where $MC_1=(r_1dx_1+r_2dx_2)/du$, and the transfer price becomes

$$r_{u_2}u_2=(1+\eta)\int\frac{dC}{du}\,du_2+\phi(u_1)$$

$$r_{u_2}=(1+\eta)[C_1(u_1+u_2)-C_1(u_1)]/u_2,$$

for $\phi(u_1) = -(1+\eta)C_1(u_1)$. Consequently, with a budget constraint the original average cost is multiplied by the opportunity profit rate $d\pi/d\bar{C}$.

As far as the divisions are concerned, costs are $(1+\eta)C_1(u)$ and $(1+\eta)C_2(q)$, and the intermediate product is transferred at an average cost. By adjusting the unit standard costs of the inputs in the indicated manner, the divisions will produce and transfer those quantities whose costs just equal the budget level. There is no need to impose direct constraints on the divisions. In fact it is likely that divisional budget constraints would be set at levels which are not in the best interests of the firm. Guidance through price adjustments should be the modus operandi of a decentralized corporation.

6. TRANSFER PRICING WITH AN ALTERNATIVE OBJECTIVE

Let us examine what happens when the firm's objective differs from profit maximization. Suppose that within a certain domain a firm is willing to trade off sales or revenues for profits, i.e., to increase revenues by a dollar the firm is willing to give up a certain amount of profit. Several such trade-off curves are shown in Figure 3, where a specific curve is defined for each level of utility. Consider, e.g., point A with revenue and profit of (R_A, π_A). Point B on the same curve has the same level of satisfaction to the firm as point A, implying that the increase in revenue $R_B - R_A$ is just worth the sacrifice in profit $\pi_A - \pi_B$.

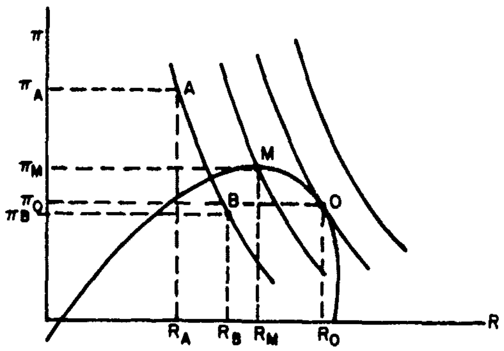

FIGURE 3

If we now sketch the profit-revenue function of the firm in the same diagram, we obtain a curve such as $F(R, \pi) = 0$.

Because the farther a trade-off curve is from the origin the higher the level of utility, the optimum occurs at point 0 where the curve F is just tangent to one of the indifference curves. The firm could maximize its profit at point M, but the trade-off curve through M has a lower utility than the trade-off curve through 0. Only when the firm does not exchange *any* increase in revenues for a dollar's worth of profit—the trade-off curves become horizontal straight lines—would the optimum occur at M.

Let us see what happens to the transfer price when a firm adopts the utility objective. We shall suppose that the divisions are designated as profit centers. If the intermediate product is traded in the market, the objectives of the divisions are the same as in section 4, while the firm's objective now becomes

maximize $U = U(R, \pi)$

subject to the constraints

$$\pi = pg(x_3, u_2) + mu_1 - r_1x_1 - r_2x_2 - r_3x_3$$

$$R = pg(x_3, u_2) + mu_1$$

$$u = f(x_1, x_2)$$

$$u = u_1 + u_2,$$

where $U(R, \pi)$ represents a utility function specifying the acceptable trade-offs between revenues and profits.

The corresponding Lagrangian functions are:

$$L_1 = mu_1 + r_{u_2}u_2 - r_1x_1 - r_2x_2 - \lambda_1[f(x_1, x_2) - u] - \theta_1[u_1 + u_2 - u]$$

$$L_2 = pg(x_3, u_2) - r_3x_3 - r_{u_2}u_2$$

$$L = U(R, \pi) - \lambda[f(x_1, x_2) - u] - \theta[u_1 + u_2 - u] - \eta[\pi - pg - mu_1 + r_1x_1 + r_2x_2 + r_3x_3] - \rho[R - pg - mu_1],$$

where λ, θ, η, ρ, etc., are Lagrangian multipliers, and x_1, x_2, x_3, u_1, u_2, u, π, R variables. If, as usual, we assume that all functions can be differentiated, and we let the market price of the final commodity be a constant, the following first-order conditions hold:

Div. 1	*Div. 2*	*Firm*
$\frac{\partial(mu_1)}{\partial u_1} - \theta_1 = 0$	$pg_3 - r_3 = 0$	$U_R - \rho = 0$
$\frac{\partial(r_{u_2}u_2)}{\partial u_2} - \theta_1 = 0$	$pg_{u_2} - \frac{\partial(r_{u_2}u_2)}{\partial u_2} = 0$	$U_\pi - \eta = 0$
$-r_1 - \lambda_1 f_1 = 0$		$(\eta + \rho)pg_3 - \eta r_3 = 0$
$-r_2 - \lambda_1 f_2 = 0$		$(\eta + \rho)pg_{u_2} - \theta = 0$
$f - u = 0$		$(\eta + \rho)\frac{\partial(mu_1)}{\partial u_1} - \theta = 0$
$u_1 + u_2 - u = 0$		$-\eta r_1 - \lambda f_1 = 0$
$\lambda_1 + \theta_1 = 0$		$-\eta r_2 - \lambda f_2 = 0$
		$f - u = 0$
		$u_1 + u_2 - u = 0$
		$\lambda + \theta = 0$
		$\pi - pg - mu_1 + r_1x_1 + r_2x_2 + r_3x_3 = 0$
		$R - pg - mu_1 = 0.$

Comparing the divisional equations with the firm's equations, we notice the following desired and actual correspondences:

$$\theta_1 = \theta/(\eta+\rho)$$

$$\lambda_1 = \lambda/(\eta+\rho)$$

$$-\lambda_1 = -\lambda/\eta = MC_1 = dC_1/du.$$

Therefore,

$$\frac{\partial(r_{u_2}u_2)}{\partial u_2} = \frac{\theta}{\eta \div \rho} = \frac{-\lambda}{\eta+\rho}$$

(from the first-order condition $\lambda + \theta = 0$ of the firm)

$$= \frac{\eta MC_1}{\eta+\rho} \quad \text{(from the last of the noted correspondences)}$$

and

$$r_{u_2}u_2 = \frac{\eta}{\eta+\rho}\int \frac{dC_1}{du}\,du_2 + \phi(u_1) = \left(\frac{U\pi}{U_\pi+U_R}\right) C_1(u_1+u_2) + \phi(u_1)$$

determines a transfer price of

$$r_{u_2} = \left(\frac{U_\pi}{U_\pi+U_R}\right)[C_1(u_1+u_2) - C_1(u_1)]/u_2.$$

Observe that when each of the input prices r_1, r_2, and r_3 is multiplied by the factor $\eta/(\eta+\rho)$ so that the standard unit input costs to the divisions become $\eta r_i/(\eta+\rho)$ $(i=1,2,3)$, each division acting as a profit center will generate the solution desired as optimal by the firm. As far as the divisions are concerned, the cost of the intermediate product is $\eta C_1/(\eta+\rho)$ after adjustment and they are transferring once more at an average cost. Although the divisions are not subjected to the direct constraints $\pi - pg - mu_1 + r_1x_1 + r_2x_2 + r_3x_3 = 0$, $R - pg - mu_1 = 0$, these equations will be automatically satisfied with the η, ρ specified at their optimal values (which are computed from the first-order conditions of the firm).

Since in our diagram $U_\pi, U_R \geqslant 0$, it follows that $U_\pi/(U_\pi+U_R) = \eta/(\eta+\rho) \leqslant 1$. Thus the effect of the adjustment is to reduce the original costs, which, of course, implies that the divisions will sell and use more of the intermediate product. Notice that when the firm is a profit maximizer and derives no satisfaction from increased sales if this means a decrease in profits, $U_R=0$, $U_\pi/(U_\pi+U_R)=1$, and we get the profit maximizing solution.

7. CONCLUSIONS

We have shown that an average cost is the correct theoretical transfer price under a variety of conditions. When constraints are included which are normally always present in any real case, the transfer price becomes an adjusted average cost. It is obviously difficult and perhaps often impossible to adjust prices precisely in practice. But if there is a commitment to the principles of transfer pricing, the approximate magnitudes of the required adjustments can be determined to give Adam Smith's invisible hand a fair chance to guide the divisions toward the optimum of the firm.

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